

The decay $\tau \rightarrow K^0 K^- \nu_\tau$ in the extended Nambu-Jona-Lasinio model

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Abstract

The full and differential widths of the decay $\tau \rightarrow K^0 K^- \nu_\tau$ are calculated in the framework of the extended Nambu-Jona-Lasinio model. The contributions of the subprocesses with the intermediate vector mesons $\rho(770)$ and $\rho(1450)$ are taken into account. The obtained results are in satisfactory agreement with the experimental data.

1 Introduction

The present paper finishes the series of works devoted to describing of τ -decays in neutrino and two pseudoscalar mesons in the framework of the extended Nambu-Jona-Lasinio (NJL) model [1, 2, 3, 4, 5]. Indeed, recently, the widths of the decays $\tau \rightarrow (\pi, \pi(1300))\nu_\tau$ [6], $\tau \rightarrow (\eta, \eta')\pi\nu_\tau$ [7], $\tau \rightarrow K^-\pi^0\nu_\tau$ [8], $\tau \rightarrow (\eta, \eta')K^-\nu_\tau$ [9] were calculated by using this model without applying any additional arbitrary parameters. This approach differs the NJL model [5, 10, 11, 12] from some other phenomenological models used for describing such processes [13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. As a rule, these models use the vector dominance method, chiral symmetry and series of arbitrary parameters which are different for various processes.

2 The Lagrangian of the extended NJL model for the mesons $K^{0,\pm}, \rho^\pm$ and their first radially excited states

In the extended NJL model, the quark-meson interaction Lagrangian for pseudoscalar $K^{0,\pm}$, vector ρ^\pm mesons and their first radially excited states takes the form:

$$\Delta L_{int}(q, \bar{q}, K, \rho) = \bar{q} \left[i\gamma^5 \sum_{j=0,\pm} \lambda_j^K (a_K K^j + b_K K'^j) + \frac{1}{2} \gamma^\mu \sum_{j=\pm} \lambda_j^\rho (a_\rho \rho_\mu^j + b_\rho \rho_\mu'^j) \right] q, \quad (1)$$

where q and \bar{q} are the u-, d- and s- constituent quark fields with masses $m_u = m_d = 280\text{MeV}$, $m_s = 420\text{MeV}$ [4],[23], $K^{0,\pm}, \rho^\pm$ are the pseudoscalar and vector mesons, the excited states are marked with prime,

$$\begin{aligned} a_a &= \frac{1}{\sin(2\theta_a^0)} \left[g_a \sin(\theta_a + \theta_a^0) + g_a' f_a(\vec{k}^2) \sin(\theta_a - \theta_a^0) \right], \\ b_a &= \frac{-1}{\sin(2\theta_a^0)} \left[g_a \cos(\theta_a + \theta_a^0) + g_a' f_a(\vec{k}^2) \cos(\theta_a - \theta_a^0) \right], \end{aligned} \quad (2)$$

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$f(\vec{k}^2) = 1 + d\vec{k}^2$ is the form factor for description of the first radially excited states [1],[2], d is the slope parameter, θ_a and θ_a^0 are the mixing angles for the strange mesons in the ground and excited states

$$\begin{aligned} d_{uu} &= -1.784\text{GeV}^{-2}, & d_{us} &= -1.761\text{GeV}^{-2}, \\ \theta_K &= 58.11^\circ, & \theta_\rho &= 81.8^\circ, \\ \theta_K^0 &= 55.52^\circ, & \theta_\rho^0 &= 61.5^\circ. \end{aligned} \quad (3)$$

The matrices

$$\begin{aligned} \lambda_+^K &= \sqrt{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_-^K &= \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda_0^K &= \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda_+^\rho &= \sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_-^\rho &= \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (4)$$

The coupling constants:

$$\begin{aligned} g_K &= \left(\frac{4}{Z_K} I_2(m_u, m_s) \right)^{-1/2} \approx 3.77, & g_K' &= \left(4I_2^{f_{us}^2}(m_u, m_s) \right)^{-1/2} \approx 4.69, \\ g_\rho &= \left(\frac{2}{3} I_2(m_u, m_u) \right)^{-1/2} \approx 6.14, & g_\rho' &= \left(\frac{2}{3} I_2^{f_{uu}^2}(m_u, m_u) \right)^{-1/2} \approx 9.87, \end{aligned} \quad (5)$$

where

$$Z_K = \left(1 - \frac{3(m_u + m_s)^2}{2M_{K_1}^2} \right)^{-1} \approx 1.83, \quad (6)$$

Z_K is the factor corresponding to the $K - K_1$ transitions, $M_{K_1} = 1272\text{MeV}$ [24] is the mass of the axial-vector K_1 meson, and the integral I_2 has the following form:

$$I_2^{f^n}(m_1, m_2) = -i \frac{N_c}{(2\pi)^4} \int \frac{f^n(\vec{k}^2)}{(m_1^2 - k^2)(m_2^2 - k^2)} \theta(\Lambda_3^2 - \vec{k}^2) d^4k, \quad (7)$$

$\Lambda_3 = 1.03 \text{ GeV}$ is the cut-off parameter [4].

The all these parameters were calculated earlier and are standard for the extended NJL model [2, 4].

3 The amplitude of the decay $\tau \rightarrow K^0 K^- \nu_\tau$ in the extended NJL model

The diagrams of the process $\tau \rightarrow K^0 K^- \nu_\tau$ are shown in Figs.1,2.

The amplitude of this process takes the form:

$$\begin{aligned} T &= -2\sqrt{2}iG_F|V_{ud}|l^\mu \left\{ I_{KK}g_{\mu\nu} + \frac{I_{KK\rho}C_\rho}{g_\rho} \cdot \frac{g_{\mu\nu}q^2 - q_\mu q_\nu}{M_\rho^2 - q^2 - i\sqrt{q^2}\Gamma_\rho} \right. \\ &\quad \left. + \frac{I_{KK\rho'}C_{\rho'}}{g_\rho} \cdot \frac{g_{\mu\nu}q^2 - q_\mu q_\nu}{M_{\rho'}^2 - q^2 - i\sqrt{q^2}\Gamma_{\rho'}} \right\} (p_{K^0} - p_{K^-})^\nu, \end{aligned} \quad (8)$$

where $G_F = 1.16637 \cdot 10^{-11}\text{MeV}^{-2}$ is the Fermi constant, $V_{ud} = 0.97428$ is the element of the Cabbibo-Kobayashi-Maskawa matrix, $l^\mu = \bar{\nu}_\tau \gamma^\mu \tau$ is the lepton current, $q = p_{K^0} - p_{K^-}$, $M_\rho =$

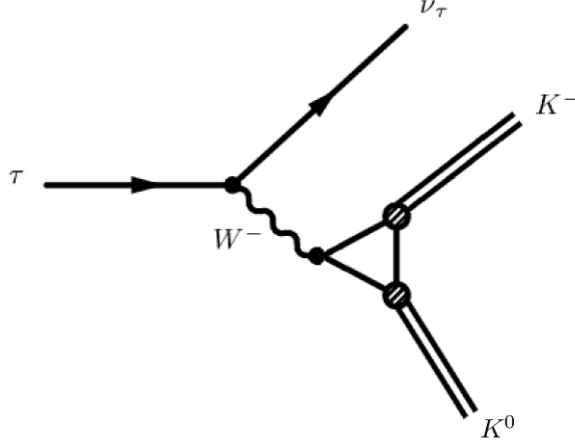


Figure 1: The decay $\tau \rightarrow K^0 K^- \nu_\tau$ with the intermediate W -boson (Contact diagram)

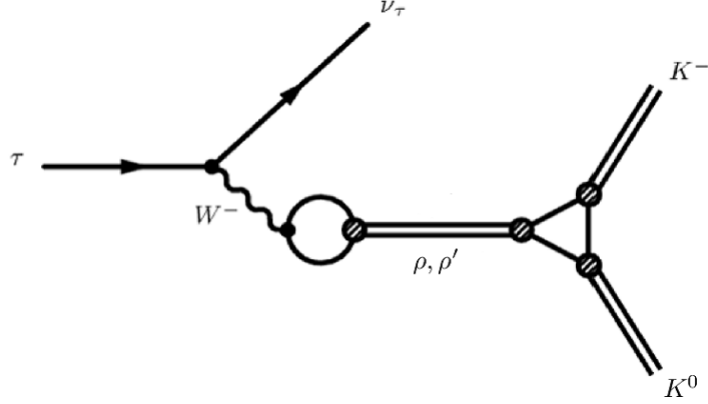


Figure 2: The decay $\tau \rightarrow K^0 K^- \nu_\tau$ with the intermediate vector $\rho(770)$ and $\rho(1450)$ mesons

$775.11 \pm 0.34 \text{ MeV}$, $M_{\rho'} = 1465 \pm 25 \text{ MeV}$, $\Gamma_\rho = 149.1 \pm 0.8 \text{ MeV}$, $\Gamma_{\rho'} = 400 \pm 60 \text{ MeV}$ are the masses and the full widths of the vector mesons [24].

The first term corresponds to the diagram with the intermediate W -boson, the second and third terms correspond to the diagrams with the intermediate vector mesons $\rho(770)$ and $\rho(1450)$. The numerical coefficients

$$\begin{aligned}
 C_\rho &= \frac{1}{\sin(2\theta_\rho^0)} \left[\sin(\theta_\rho + \theta_\rho^0) + R_V \sin(\theta_\rho - \theta_\rho^0) \right], \\
 C_{\rho'} &= \frac{-1}{\sin(2\theta_\rho^0)} \left[\cos(\theta_\rho + \theta_\rho^0) + R_V \cos(\theta_\rho - \theta_\rho^0) \right], \\
 R_V &= \frac{I_2^f(m_u, m_u)}{\sqrt{I_2(m_u, m_u) I_2^{f^2}(m_u, m_u)}}. \\
 I_{abc} &= -i \frac{N_c}{(2\pi)^4} \int \frac{a(\vec{k}^2) b(\vec{k}^2) c(\vec{k}^2)}{(m_s^2 - k^2)(m_u^2 - k^2)} \theta(\Lambda_3^2 - \vec{k}^2) d^4 k,
 \end{aligned} \tag{9}$$

where $a(\vec{k}^2)$, $b(\vec{k}^2)$ and $c(\vec{k}^2)$ are the coefficients from the Lagrangian defined in (2).

4 Numerical estimations

The calculated branching of the process $\tau \rightarrow K^0 K^- \nu_\tau$ is

$$Br(\tau \rightarrow K^0 K^- \nu_\tau) = 12.7 \cdot 10^{-4}. \quad (10)$$

The experimental value of this branching are

$$Br(\tau \rightarrow K^0 K^- \nu_\tau)_{exp} = (14.9 \pm 0.5) \cdot 10^{-4} \quad [24] \quad (11)$$

$$Br(\tau \rightarrow K^0 K^- \nu_\tau)_{exp} = (15.1 \pm 4.3) \cdot 10^{-4} \quad [25]. \quad (12)$$

The mass and full decay width of the meson $\rho(1450)$ are not defined precisely. It is interesting to note that if we choose the minimal values of mass and full width of this meson ($M_{\rho'} = 1440\text{MeV}$, $\Gamma_{\rho'} = 340\text{MeV}$), the results are in better agreement with the experimental data:

$$Br(\tau \rightarrow K^0 K^- \nu_\tau) = 14.7 \cdot 10^{-4}. \quad (13)$$

The comparison of the calculated and experimental differential width is shown in Fig. 3. The solid lines correspond to our theoretical differential width. The blue one is for the case of middle values of mass and full width of the meson $\rho(1450)$, the red one is for the case of minimal values of them. The points correspond to the experimental values [25].

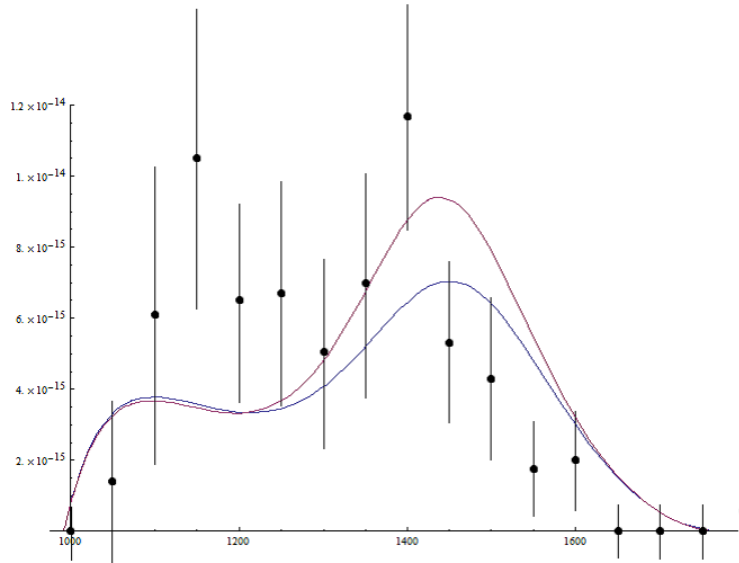


Figure 3: Differential width of the decay $\tau \rightarrow K^0 K^- \nu_\tau$

5 Conclusion

From two options considered in this paper, it is easy to see that the results are sensitive to the choice of the full decay width and mass of $\rho(1450)$ meson. Since these mass and width are defined with the large errors, we can choose the minimal allowable values. This leads to better agreement with the experimental data. It is interesting to compare our results with the results from other phenomenological models and experiments. Such comparison is shown in Tab. 1. Our results are in satisfactory agreement with the experimental data.

Table 1: The branching ratios for the process $\tau \rightarrow K^0 K^- \nu_\tau$.

	Br ($\times 10^{-4}$)	References
Theory	27	B.A. Li [14]
	12.5 ± 1.3	J.E. Palomar [15]
	13.5/19	H.Czyz, A. Grzelinska, J.H. Kuhn [17]
	16	S. Dubnicka, A.Z. Dubnickova [18]
	12.7(14.7)	Our result
Experiment	15.1 ± 4.3	CLEO [25]
	16.2 ± 3.2	ALEPH [26]
	14.8 ± 0.68	Belle [27]
	14.9 ± 0.5	PDG [24]

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